Issue Date: 21.01.2013
Version: 2013-01-21 15:37

Exercise Sheet 7 zur Vorlesung Computational Biology (Part 2), WS 12/13

Hand In: *Until Monday,* ***28.01.2013****, 10:00 am, email to* `wild@cs...` *or in lecture.*

Exercise 14

1 + 1 + 1 + 1 + 3 Points

Consider Zuker's algorithm as given on pages 201f in the lecture script.

a) Show that the recursion for $E(i, j)$ can be equivalently written as

$$E(i,j) := \min \begin{cases} E(L_{i,j}) \\ \min\limits_{i \leq k < j} E(i,k) + E(k+1,j) \end{cases} \tag{1}$$

(Note the $\leq$ instead of the $<$ in the inner minimum!)

b) Show that E fulfills a kind of *triangle inequality*:

$$\forall i \leq k < j \ : \ E(i,j) \leq E(i,k) + E(k+1,j)$$

c) Consider again recurrence (1) and assume that the value of $E(i, j)$ resulted from the second alternative, i. e. formally $\exists k : E(i, j) = E(i, k) + E(k + 1, j)$. Let k be *minimal* with this property.

Show that $E(i, k) = E(L_{i,k})$, i. e. the minimum for computing $E(i, k)$ was attained by the first alternative in (1).

Note: This means that in the bifurcation alternative, we only need to consider split points k, where the optimal substructure for range $i \dots k$ includes the base pair (i, k)!

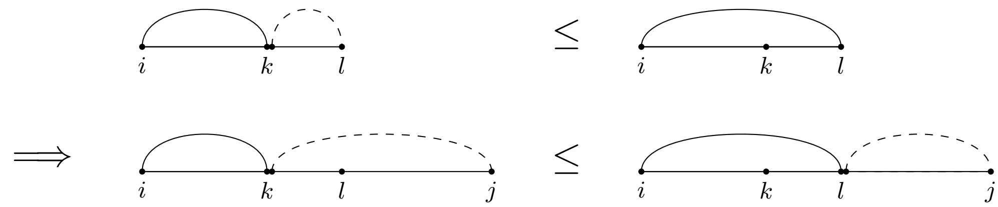
d) Prove the following *domination relation*:

Let $i < k \leq l$ be indices such that $E(L_{i,k}) + E(k+1, l) \leq E(L_{i,l})$. Then for any $j \geq l$ also

$$E(L_{i,k}) + E(k+1, j) \leq E(L_{i,l}) + E(l+1, j)$$

holds.

For the more visually inclined, the claim says



Note: The dominance relation says that if for substructure $i \ldots l$, including base pair (i, l) did not improve energy, neither does it when we extend the substructure to the right.

e) Use the results of c) and d) to design a variant of Zuker's algorithm that does *not* naïvely iterate over all possible values for k in the bifurcation alternative.