TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Department of Computer Science
Sebastian Wild

Issue Date: 28.06.2017
Version: 2017-06-28 16:59

10th Exercise sheet for Advanced Algorithmics, Summer 17

Hand In: *Until Wednesday, 05.07.2017, 12:00 am, hand-in box in 48-4 or via email.*

Please participate in the course evaluation!
`https://vlu.informatik.uni-kl.de/`

Problem 23

30 points

Prove Theorem 5.8:
`greedyMaxCut` is a (deterministic) 2-approximation for MAX-CUT.

N. B.: *Lecture Notes 18 have been updated to show the correct algorithm.*

Problem 24

20 points

Show that there is no $\epsilon > 0$ so that `layeringSetCover` is an $(f - \epsilon)$-approximation for SET-COVER, i. e. that f is tight.

Hint: Give a set of instances that contains infinitely many counterexamples for every $\epsilon > 0$.

Problem 25

30 points

We consider the vertex cover problem again, which can be written as the following ILP:

$$\begin{aligned} \min \quad & \sum_{v \in V} x_v \\ \text{s. t.} \quad & x_u + x_v \geq 1 \quad \text{for all } \{u, v\} \in E \\ & x_v \in \mathbb{N}_0 \quad \text{for all } v \in V \end{aligned}$$

a) Argue that any *optimal* solution x^* to above ILP "is 0/1", i.e., consists only of entries 0 and 1.

b) Consider the LP relaxation of the vertex cover ILP and determine its dual LP.

c) Now consider the dual LP restricted to integer solutions; argue that here in fact *any* feasible integral solution is 0/1.

   Which graph problem is described by the dual LP with integrality constraints?

d) Assume you are given an optimal integral solution for the dual LP. What do the complementary slackness conditions allow to deduce about an optimal solution of the primal LP?