

7th Exercise sheet for Advanced Algorithmics, Summer 17

Hand In: Until Wednesday, 14.06.2017, 12:00 am, hand-in box in 48-4 or via email.

Problem 18

30 + 30 points

- a) Give detailed pseudocode for insert and delete operations in treaps.

Assume that upon insertion, you are already given a key-priority pair.

- b) Prove Lemma 4.39 (Bound on Rotations) from class:

The number of rotations to insert or delete a node x in a randomized treap is at most $LS(x) + RS(x)$, where $LS(x)$ and $RS(x)$ are the lengths of the left resp. right spine of (the subtree of) x in the treap (after insertion resp. before deletion).

Hint: Start with deletion; consider the effect of one rotation.

Problem 19

30 points

Devise an algorithm that splits treaps S as efficiently as possible into two treaps $S_{\leq k}$ and $S_{>k}$ which contain all keys in S that are $\leq k$ resp. $> k$.

Now also devise the inverse algorithm that joins two treaps $S_{\leq k}$ and $S_{>k}$ where $S_{\leq k}$ contains only keys $\leq k$ and $S_{>k}$ contains only keys $> k$ into a single treap.

Argue why your algorithm is correct and what runtime it has.