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Exercise Sheet 3 for Computational Biology (Part 1), SS 15

Hand In: *Until Tuesday, 09.06.2015, 10:00 am, email to* `r_muelle@cs...`, *hand-in box in stairwell 48-6 or in lecture.*

Problem 7

3 + 6 points

a) Design an algorithm for computing an optimal global alignment score (only the score, not the alignment itself!) with linear space complexity.

   To simplify notation, we assume that $g \geq 0$, $goal_\delta = \min$ and $\delta(a, b) \geq 0$ for all symbols $a, b \in \Sigma$. Moreover, we assume $n \leq m$.

   Formally then, for two strings S, $T \in \Sigma^\star$ of lengths m and n, respectively, your algorithm should compute $sim_\delta(S, T)$ in time $\mathcal{O}(mn)$ and space $\mathcal{O}(n)$.

b) Design an algorithm for computing an optimal global alignment in time $\mathcal{O}(mn)$ and space in $\mathcal{O}(n + \log(m))$.
   (Yes, this time it is not only the score, but the actual alignment.)

   Less efficient solutions yield partial credit.

   Hint: Use divide and conquer and a).

Problem 8

3 points

Prove that the decision version of multiple alignments with SP scoring is $\mathcal{NP}$-complete.

Hint: Try a reduction of the Dec-$(0, 1)$-Shortest-Superseq-Problem defined in the German lecture notes, page 85.

Problem 9

3 points

Prove Lemma 12 of the German lecture notes; restated here for convenience:

Given a directed graph $G = (V, E)$ with *edge weights* $g : E \to \mathbb{R}_{\geq 0}$. Define $d_g(u, v)$ to be the shortest path distance of u and v w. r. t. g, i. e. the minimum of the sum of edge weights over all paths from u to v in G. Assume further that we are given a special *target node* $t \in V$ and a *node potential* $\xi : V \to \mathbb{R}_{\geq 0}$, such that

$$\forall v \in V : \xi(v) \leq d_g(v, t) \ .$$

If we furthermore have $g(u, v) \geq \xi(u) - \xi(v)$ for all edges $(u, v) \in E$, we can define modified edge weights

$$g^*(u, v) \ := \ g(u, v) - \big(\xi(u) - \xi(v)\big) \ ,$$

which are again nonnegative. Show that *every* shortest path from u to v w. r. t. g^* is a shortest path from u to v w. r. t. g, as well.

Remark: In a geometric setting, one may think of ξ as the "straight-line distance" between two points, whereas the graph distance d_g is the distance on a road network. The sanity condition then says that a road segment is at least as long as the difference in straight-line distance to t.