

## 6th Exercise sheet for Advanced Algorithmics, SS 15

**Hand In:** Until Monday, 01.06.2015, 12:00am, in lecture, exercise sessions, hand-in box in stairwell 48-6 or via email.

### Problem 12

We have seen two different definitions of Las Vegas algorithms in class. Show that they are equivalent in a complexity-theoretic sense, that is

$$\begin{aligned} & P \text{ can be solved by an } LV_a \text{ algorithm in expected time } \Theta(f) \\ \iff & P \text{ can be solved by an } LV_b \text{ algorithm in expected time } \Theta(f) \end{aligned}$$

for some problem  $P$  and some function  $f : \mathbb{N} \rightarrow \mathbb{N}$ .

### Problem 13

Show that any one-way OSE-MC algorithm for  $\text{Equality}_n$  has communication cost of at least  $n$  (bits).

**Hint:** For partial virtual credit, show the bound in a simpler setting; assume one of  $C_1$  and  $C_2$  has to be deterministic.

### Problem 14

Give a TSE-MC-algorithm for  $\text{Equality}_n$  with communication complexity in  $\mathcal{O}(\log n)$ . Show that your algorithm has the necessary properties.

**Note:** You may assume that  $n$  is sufficiently large, that is your algorithm may violate the TSE-MC restrictions for finitely many  $n$ .

**Problem 15**

How do you construct a decider for  $L$  given an OSE-MC-algorithms for  $L$  and  $\bar{L}$ , respectively? Justify your answer.

**Problem 16**

- a) Give algorithm  $A$  that generates random permutations of the numbers  $1, \dots, n$ . Each permutation is to have the same probability.

Show that your algorithm has the desired property and determine  $\text{Exp-Time}_A(n)$  as well as  $\text{Random}_A(n)$ .

- b) Which of the classes of randomized algorithms known from lecture does  $A$  belong to?