3rd Exercise sheet for
Advanced Algorithmics, SS 15

Hand In: *Until Monday, 11.05.2015, 12:00 am, in lecture, exercise sessions, hand-in box in stairwell 48-6 or via email.*

Problem 5

Which of the following reduction rules for MAX-SAT are valid? Explain why your answers are correct.

  i) If φ contains a clause with only one literal, set the corresponding variable to the satisfying truth value, delete the clause and decrement k by 1.

 ii) If variable x occurs only positively in φ, set x to TRUE, decrement k by the number of therewith fulfilled clauses and delete these.

iii) If φ contains clauses (x) and $(\neg x)$, delete both and decrement k by 1.

 iv) If variables x, y and z occur only in a subformula

$$(x \vee y) \wedge (\neg y \vee z) \wedge (\neg x \vee \neg z)$$

of φ, delete all three clauses and decrement k by 3.

  v) If variable x occurs only in a subformula

$$(x \vee y) \wedge (y \vee z) \wedge (\neg x)$$

of φ, substitute x with y and leave k unchanged.

 vi) If variable x occurs only in a subformula

$$(x \vee y) \wedge (y \vee z) \wedge (\neg x)$$

of φ, substitute x with $\neg y$, decrement k by 1 and delete clause $(x \vee y)$.

Problem 6

Solve the following inhomogenous linear recurrence equations using generating functions:

a) $a_0 = 4,$
$$a_{i+1} = 2a_i + 3^i \quad , i \geq 0 .$$

b) $b_0 = 2,$
$b_1 = 2,$
$$b_i = 6 \cdot b_{i-1} - 8 \cdot b_{i-2} + 13 \cdot i \quad , i \geq 2 .$$

Problem 7

In lecture, we have seen a procedure to determine optimal vertex covers in time $\mathcal{O}(1.33^k)$ using search trees.

a) Show that case 3 (regular graph) occurs at most three times in every branch!

b) If case 3 could occur arbitrarily often, how large would the largest search trees be?

c) If only the number of 2-regular graphs was bounded, how large would the largest search trees be?