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Fachbereich Informatik
AG Algorithmen und Komplexität

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11th Exercise Sheet for Kombinatorische Algorithmen, WS 14/15

Hand In: *Until Monday, 02.02.2015, 12:00,*
deliver or email to Raphael (`reitzig@cs.uni-kl.de`*).*

Problem 20

2 points

Prove Theorem 6.1 by Duchon et al. [1], that is show that

$$\Pr\left[N \in n(1 \pm \varepsilon)\right] \;\to\; 1 \qquad \text{as } n \to \infty\,,$$

where N is the random size of an object returned by $\mu C(x_n; n, \varepsilon)$.
Here, we write $n(1 \pm \varepsilon)$ for short and mean the interval $\left[n(1-\varepsilon), n(1+\varepsilon)\right] \subset \mathbb{R}$.

Problem 21

$2 + 1 + 2$ points

Consider once again the class $\mathcal{S}$ of RNA secondary structures, given by

$$\mathcal{S} \;=\; \epsilon \;+\; \mathcal{Z}_{*} \times \mathcal{S} \;+\; \mathcal{Z}_{(} \times \mathcal{S} \times \mathcal{Z}_{)} \times \mathcal{S}\,, \tag{1}$$

and the Boltzmann sampler $\Gamma S(x)$ you built in Problem 19. In this exercise, we will tweak your sampler for efficiency.

a) Check whether the Boltzmann model of RNA secondary structures fulfills the *Mean Value Condition* and the *Variance Condition* [1, equations (6.1) and (6.3)].

   What guarantees does Theorem 6.1 of the same article provide for your sampler?

b) Determine the singular exponent $-\alpha$ for $S(z)$ as defined in Section 6.2 [1].

   What guarantees does Theorem 6.3 of the same article provide for your sampler?

c) Design a linear time approximate size Boltzmann sampler $\mu S(x; n, \varepsilon)$; remember to prove your claims.

   Extend your implementation from Problem to incorporate this sampling algorithm. For simplicity, you may fix $n = 100$ and precompute all necessary constants externally.

   Use your sampler to draw 10 random RNA structures of size *exactly* 100. How many rejections did your sampler need until it found an object of correct size?

References

[1] Philippe Duchon et al. “Boltzmann Samplers for the Random Generation of Combinatorial Structures.” English. In: *Combinatorics, Probability and Computing* 13.4-5 (July 2004), pp. 577–625. ISSN: 1469-2163. DOI: 10.1017/S0963548304006315.