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8th Exercise Sheet for Kombinatorische Algorithmen, WS 14/15

Hand In: *Until Monday, 12.01.2015, 12:00,*
deliver or email to Raphael (`reitzig@cs.uni-kl.de`).

The exercises on this sheet are intended to make you familiar with the use of the *symbolic method* to specify combinatorial classes. For all exercises of type "Specify ..." you are expected to answer the following questions:

- Do we deal with labeled or unlabeled atoms?
- Which atoms do we need and what sizes should they have?
- How can the given structures be constructed (recursively?) from smaller parts?
  - Briefly describe your idea. Pictures are very welcome!
  - Make sure that the specification is complete and unambiguous, i. e. every object has a *unique* construction.
  - Give the (system of) symbolic equation(s).
- What is the generating function for the class, i. e. for the sequence of numbers of objects of each size?

Problem 13

1 + 2 + 1 points

a) Give a formal specification of the class $\mathcal{T}$ of (directed, rooted) trees, in which every node has either 0, 1 or 2 children. In case of a binary node, the order of the subtrees matters (like for binary search trees). We assume nodes of the same group (same degree) to be indistinguishable (unlike for binary search trees!).

   The size $|t|$ of a tree $t \in \mathcal{T}$ is the total number of nodes in t.

b) Specify the class $\mathcal{P}$ of returning random walks with steps $\nearrow$, $\rightarrow$ and $\searrow$, i. e. paths on the two-dimensional grid $\mathbb{Z}^2$ starting in $(0,0)$ and ending in $(n,0)$ $(n \in \mathbb{N})$. Each single step of such a path can be either the vector $(1,1)$, $(1,0)$ or $(1,-1)$. Moreover, the path may never cross the x-axis, i. e. when the current point is $(k,0)$, we may not make a step $(1,-1)$.

   The size of a path is the number of steps, or equivalently its length in x-direction.

   Hint: Decompose paths according to the first step.

c) Consider the class $\mathcal{T}$ from a) again, with one difference: this time, we define $|t|$ to be the number of *edges* in the tree. Adapt your specification accordingly.

What can you say about the generating function and thus about the number T_n of trees $t \in \mathcal{T}$ with n edges?

Problem 14

1 + [3] points

a) Specify the class $\mathcal{S}_r$ of *surjective* functions f from $\{1, \ldots, n\}$ onto $\{1, \ldots, r\}$ for fixed parameter $r \in \mathbb{N}$. The size of a surjection is the size its domain, i. e. $|f| = n$.

b) **Optional Exercise:**
Specify the class $\mathcal{F}$ of (arbitrary) functions f from $\{1, \ldots, n\}$ onto set $\{1, \ldots, n\}$ with (arbitrary) size $|f| = n$.

Problem 15

3 points

Specify the class $\mathcal{B}$ of bitstrings $b \in \{0, 1\}^\star$ with the following properties:

- b ends with the pattern $P = 01001$.
- P does not occur earlier in b.

Use the number of bits in b as its size.

Hint: Remember the string matching automata we used for string matching. Can you find a specification for the bitstrings for which the automaton ends up in a certain state q_i?