TECHNISCHE UNIVERSITÄT
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Fachbereich Informatik
AG Algorithmen und Komplexität

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3rd Exercise Sheet for Kombinatorische Algorithmen, WS 14/15

Hand In: *Until Monday, 24.11.2014, 12:00,*
deliver or email to Raphael (`reitzig@cs.uni-kl.de`*).*

Problem 4

$4 + 1 + 2$ points

Recall the notion of *exhaustive history* as introduced by Lempel and Ziv [1, p 76]. We call the corresponding decomposition of the input word w *LZ77-decomposition (of w).*

Furthermore, Ziv and Lempel define a restricted variant of this decomposition in their proof of Theorem 2 in a later work [2, p 533]. We consider the limit for $n \to \infty$ and call the result *LZ78-decomposition.*

a) Give formal definitions of the LZ77- and LZ78-decomposition for arbitrary $w \in \Sigma^\star$, respectively. Use our notation.

   What are similarities and differences between the two?

b) Give

   (i) the LZ77-decomposition,

   (ii) the LZ78-decomposition

   (iii) and an arbitrary *non*-exhaustive history

   of $w = aaaaabbababaaabb$.

c) Prove:

   (i) Every $w \in \Sigma^\star$ has exactly one LZ77-decomposition.

   (ii) Every $w \in \Sigma^\star$ has exactly one LZ78-decomposition.

Problem 5

2 + 2 points

Prove the following *no-free-lunch* theorems for lossless compression.

a) For every compression algorithm A there is an input $w \in \Sigma^\star$ for which $|A(w)| \geq |w|$, i. e. the "compression" is no shorter than the input.

b) For every compression algorithm A and $n \in \mathbb{N}$,

$$\left|\{w \in \Sigma^{\leq n} : |A(w)| < |w|\}\right| < \frac{1}{2} \cdot \left|\Sigma^{\leq n}\right| ,$$

that is less than half of all inputs of length at most n can be compressed below their original size.

As domain of (all) compression algorithms, we consider the set of (all) injective functions in $\Sigma^\star \to \Sigma^\star$.

The theorems hold for every non-unary alphabet; you can restrict yourself to the binary case, i. e. $\Sigma = \{0, 1\}$, though.

References

[1] Abraham Lempel and Jacob Ziv. "On the Complexity of Finite Sequences." In: *Information Theory, IEEE Transactions on* 22.1 (1976), pp. 75–81. DOI: 10.1109/TIT.1976.1055501.

[2] Jacob Ziv and Abraham Lempel. "Compression of individual sequences via variable-rate coding." In: *Information Theory, IEEE Transactions on* 24.5 (1978), pp. 530–536. DOI: 10.1109/TIT.1978.1055934.