TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

TU Kaiserslautern
AG Algorithmen und Komplexität

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1st Exercise Sheet for Kombinatorische Algorithmen, WS 14/15

Hand In: *Until Monday, 10.11.2014, 12:00,*
deliver or email to Raphael (`reitzig@cs.uni-kl.de`).

Problem 1

3 + 2 points

Let $P \in \Sigma^m$. Prove the following properties of prefix function Π_P.

a)

$$\Pi_P^\star(q) \;=\; \{k \mid k < q \;\wedge\; P_{0,k} \sqsupset P_{0,q}\} \qquad \text{for all } q \in \{1, 2, \ldots, m\}\ ,$$

where $\Pi_P^\star$ is the iterated prefix function (cf. Definition 6.5 in [Neb12b]).

b)

$$\Pi_P(q) \;=\; \begin{cases} 0 & \text{wenn } E_{q-1} = \emptyset \\ 1 + \max\{k \in E_{q-1}\} & \text{wenn } E_{q-1} \neq \emptyset \end{cases}$$

for all $q \in \{2, 3, \ldots, m\}$, where

$$E_q := \{k \mid k \in \Pi_p^\star(q) \wedge P_{k+1} = P_{q+1}\}$$

for $q \in \{1, 2, \ldots, m-1\}$.

Problem 2

4 points

Develop a linear-time algorithm for the following problem:

Input: $A, B \in \Sigma^n$ with $n \in \mathbb{N}$ and some alphabet Σ.

Question: Is there a $k \in \mathbb{N}_0$ so that

$$A_{i+1} \;=\; B_{s(i)+1} \qquad \text{with } s(i) := (i + k) \bmod n\ .$$

holds for all $i \in [0..n-1]$?

Determine an infinite family of worst-case inputs for your procedure, e. g. by giving a scheme depending on n. How many symbol comparisons does your algorithm need on these inputs?