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Exercise Sheet 10 for Algorithm Engineering, SS 14

Hand In: *Until Monday, 07.07.2014, 10:00 am, email to* `wild@cs...` *or in lecture.*

Problem 23

3 points

Recall the method from class for precisely solving the mergesort recurrence

$$f_n \;=\; f_{\lfloor n/2 \rfloor} + f_{\lceil n/2 \rceil} + e_n \;.$$

The main trick consisted in using forward-backward differences $\Delta\nabla f_n$ to get rid of the floor and ceiling functions and then consider the Dirichlet generating function of $\Delta\nabla f_n$. The subsequent computations involved the Dirichlet generating function of the forward-backward differences of the toll function

$$\boxminus(s) \;=\; \sum_{k\geq 1} \frac{\Delta\nabla e_k}{k^s} \,,$$

which we only computed for the worst-case costs.

Give representation of $\boxminus(s)$ — as simple as possible — for

a) the best case, i. e., for $e_n = \lfloor \frac{n}{2} \rfloor$, and for

b) the average case, i. e., with $e_n \;=\; n - \dfrac{\lfloor n/2 \rfloor}{\lceil n/2 \rceil + 1} - \dfrac{\lceil n/2 \rceil}{\lfloor n/2 \rfloor + 1}$.

Problem 24

3 points

Compute the following integral using the residue theorem

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx \;.$$