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Exercise Sheet 7 for Algorithm Engineering, SS 14

Hand In: *Until* ***Tuesday, 10****.06.2014, 12:00 am, email to* `wild@cs...` *or in lecture.*

Problem 16

2 points

Let $L_1 \subseteq \Sigma_1^*$, $L_2 \subseteq \Sigma_2^*$ be two languages over disjoint alphabets $\Sigma_1 \cap \Sigma_2 = \emptyset$ and let $L = L_1 \sqcup\!\sqcup L_2$ be their shuffle product. Moreover, denote by $\hat{G}$, $\hat{G}_1$ and $\hat{G}_2$ the *exponential* generating functions of L, L_1 and L_2, resp. Show that it holds

$$\hat{G}(z) \;=\; \hat{G}_1(z) \cdot \hat{G}_2(z)\ .$$

Problem 17

8 points

We consider a cache with the *least-recently-used (LRU)* eviction strategy. There are m pages $a_1, \ldots, a_m$ in total that can be accessed and the cache can hold (at most) k pages at any time. Assume that the accesses to pages are i. i. d. where a_i is requested with probability for p_i. Show that the probability D of a cache-miss satisfies

$$1 - D \;=\; \sum_{q=0}^{k-1} [u^q] \int_0^\infty \Phi(t,u)\Psi(t,u)\, e^{-t}\, dt,$$

where

$$\begin{aligned}\Phi(t,u) &= \prod_{i=1}^{m} (1 + u(e^{p_i t} - 1)),\\ \Psi(t,u) &= \sum_{i=1}^{m} \frac{p_i^2}{1 + u(e^{p_i t} - 1)},\end{aligned}$$

Hint: You may use without proof, that

$$\sum_{q=0}^{k-1} [u^q] \int_0^\infty e^{-t} \prod_{i=1}^{m-1} \left(1 + u(e^{p_i t z} - 1)\right) dt$$

is analytic for $|z| \leq 1$.