Exercise Sheet 3 for
Algorithm Engineering, SS 14

Hand In: *Until Monday, 12.05.2014, 10:00 am, email to* `wild@cs...` *or in lecture.*

Problem 5 $\qquad$ 3 points

We consider randomized skiplists with unbounded height of the pointer towers. Prove that for r the maximal height actually occuring holds

$$\mathbf{Pr}[r > \alpha \log_2 n] \ \leq \ \frac{1}{n^{\alpha-1}}.$$

Problem 6 $\qquad$ 2 + 1 points

Let X be a binomially distributed random variable with parameter p and size n.

   a) Find a closed form of the probability generating function $P(z)$ for X.

   b) Using the probability generating function determine expectation and variance of X.

Problem 7 $\qquad$ 3 points

Compute a closed form representation of the series $(a_n)_{n\in\mathbb{N}}$ defined by

$$a_0 \ = \ 1$$
$$a_n \ = \ -2a_{n-1} + n + 3, \qquad \text{for } n \geq 1,$$

using generating functions.

Hint: Start by transforming the recurrence equation for a_n into a functional equation for the ordinary generating function $A(z)$ for $(a_n)_{n\in\mathbb{N}}$. Then solve this functional equation to get a closed form for $A(z)$ and try to use known series formulas to obtain $[z^n]A(z)$ from it; computing the *partial fraction decomposition* of $A(z)$ is a very helpful intermediate step in doing so. A comprehensive collection of helpful formulas is given on the *Theoretical Computer Science Cheat Sheet*: `http://www.tug.org/texshowcase/cheat.pdf`.