TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Computer Science
Algorithms & Complexity Group

Issue Date: 28.04.2014
Version: 2014-04-28 18:58

Exercise Sheet 2 for Algorithm Engineering, SS 14

Hand In: *Until Monday, 05.05.2014, 10:00 am, email to* `wild@cs...` *or in lecture.*

Problem 2

2 + 2 points

Let γ be a (closed) path that (once) traverses the unit circle counter-clockwise. Compute the following contour integrals using residue calculus.

You may use computer algebra to compute derivatives and for term simplification, but full credit is given only for solutions with sufficiently detailed intermediate steps.

a) $\displaystyle\oint_\gamma \frac{z^5+1}{2z^4-5z^3+2z^2}$,

b) $\displaystyle\oint_\gamma \frac{2iz^3-5z+i}{256z^4-96z^2+9}$.

Problem 3

1 + 2 + 2 + 1 points

In this exercise, we use the elementary formal definition of complex contour integrals to derive an alternative proof of the *residue theorem* from class.

Let $G \subset \mathbb{C}$ be an open, contiguous complex region and $f : G \to \mathbb{C}$ a continuous complex function. Moreover, let $\gamma : [0,1] \to \mathbb{C}$ be a complex-valued, continuously differentiable function. Then, the *contour integral* of f along the *path of integration* γ is defined as

$$\int_\gamma f(z)\,dz \quad := \quad \int_0^1 f\bigl(\gamma(t)\bigr)\gamma'(t)\,dt$$

where the latter is understood as an ordinary real-valued integral for real and imaginary part separately, i. e.,

$$\int_0^1 f(\gamma(t))\gamma'(t)\,dt \;=\; \int_0^1 \Re\Big(f(\gamma(t))\gamma'(t)\Big)\,dt \;+\; i\int_0^1 \Im\Big(f(\gamma(t))\gamma'(t)\Big)\,dt\,.$$

That way, we have reduced all complex integrals to the well-known ones from real calculus.

a) The most common case of contours γ (for us) is a closed path describing a counter-clockwise circle with radius $r \in \mathbb{R}_{>0}$ around the origin. Give an explicit term for γ that describes such a circle. Compute its derivative $\gamma'(t)$.

b) Using the contour from a), find a continuous complex function f, such that

$$f(\gamma(t)) \cdot \gamma'(t) \;=\; 1$$

for all $t \in [0, 1]$ and compute $\int_\gamma f(z)\,dz$ using the formal definition of the contour integrals from above.

c) Compute $\int_\gamma z^n\,dz$ for all $n \in \mathbb{Z}$ and γ from a).

d) Using the result from c), re-derive the following fact (used in the proof idea of the residue theorem) without (explicit) use of complex antiderivatives:

$$\int_\gamma \sum_{k=-\infty}^{\infty} a_k z^k\,dz \;=\; 2\pi i \cdot a_{-1}\,.$$

Problem 4

2 + 3 points

We consider the dynamic arrays from lecture whose contents are copied to a larger new array if more elements arrive. Determine

1. the worst and
2. average case *unused memory*
3. as well as the number of copied elements

for arrays of dynamic size if the following extension strategies are used:

a) The array is extended by a fixed amount δ each time.

b) The new size of the array is the sum of the previous two sizes. For the first extension the size of the array is doubled.