TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

TU Kaiserslautern
AG Algorithmen und Komplexität

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Exercise Sheet 11 for Combinatorial Algorithms, SS 13

Hand In: *Until Monday, 15.07.2013, 12:00,*
box in the group's hallway or email to `wild@cs.uni...`.

Problem 19

2 points

Prove Theorem 6.1 of [DFLS04], i. e., show that

$$\Pr\big[N \in n(1 \pm \varepsilon)\big] \;\to\; 1 \qquad \text{as } n \to \infty\,,$$

where N is the random size of an object returned by $\mu C(x_n; n, \varepsilon)$.

Here, we briefly write $n(1 \pm \varepsilon)$ to mean the interval $\big(n(1-\varepsilon), n(1+\varepsilon)\big) \subset \mathbb{R}$.

Problem 20

$2 + 1 + 2$ points

Consider once again the class $\mathcal{S}$ of RNA secondary structures, given by

$$\mathcal{S} \;=\; \epsilon \;+\; \mathcal{Z}_{*} \times \mathcal{S} \;+\; \mathcal{Z}_{(} \times \mathcal{S} \times \mathcal{Z}_{)} \times \mathcal{S}\,, \tag{1}$$

and the Boltzmann sampler $\Gamma S(x)$ you built in Problem 18. In this exercise, we will tweak your sampler for efficiency.

a) Check whether the Boltzmann model of RNA secondary structures fulfills the *Mean Value Condition* and the *Variance Condition* stated in equations (6.1) and (6.3) of [DFLS04]. What does Theorem 6.1 guarantee for your Boltzmann sampler?

b) Determine the singular exponent $-\alpha$ for $S(z)$ as defined in Section 6.2 of [DFLS04]. What does Theorem 6.3 of [DFLS04] guarantee about your Boltzmann sampler?

c) Design a linear time approximate size Boltzmann sampler $\mu S(x; n, \varepsilon)$. Don't forget to prove your claims.

   Extend your implementation from Problem 18 to incorporate this sampling algorithm. For simplicity, you may fix $n = 100$ and compute all needed constants externally up front.

   Use your sampler to draw 10 random RNA structures of size *exactly* 100. How many rejections did your sampler need until it found an object of correct size?

References

[DFLS04] Philippe Duchon, Philippe Flajolet, Guy Louchard, and Gilles Schaeffer. Boltzmann Samplers for the Random Generation of Combinatorial Structures. *Combinatorics, Probability and Computing*, 13(4-5):577–625, July 2004. doi:10.1017/S0963548304006315.