Exercise Sheet 6 for Combinatorial Algorithms, SS 13

Hand In: *Until Monday, 10.06.2013, 12:00,*
box in the group's hallway or email to `wild@cs.uni...`.

Problem 9

2 points

The *max-flow-min-cut theorem* relates the flow values of maximal flows and the capacities of minimal cuts: The optimal solution *values* of both problems coincide. However, additional work is needed to compute the actual *solutions* — and we only considered flow algorithms in class.

Design an algorithm for computing a minimum capacity (s,t)-cut in the network $G = (V,E)$ from an optimal flow f^* in G.

Problem 10

5 points

The *augmenting path algorithm* is related to the *primal* simplex algorithm[1] for solving *LPs (linear programs)*: We start with and maintain a feasible solution, which is initially suboptimal. This solution is successively improved until we reach optimality.

In this exercise, we show that the *preflow push algorithm* resembles the *dual* simplex method: It maintains a *dually* feasible solution, which in our case is a (s,t)-cut. This dual solution is modified until it becomes (primally) feasible; here until the preflow becomes a flow.

The original statement of the preflow push algorithm does not explicitly maintain this cut, therefore we augment the algorithm as follows:

We maintain a set S of nodes throughout the algorithm, which is initially $S := \{s\}$. The algorithm then invokes a sequence of relabel and push operations. Each push moves some amount of flow over an edge $(u,v) \in E_f$ of the *residual* network G_f. After each such push-operation, we now check whether $u \notin S \wedge v \in S$, i. e. whether the push was "into S". If yes, we add to S all nodes reachable from u in G_f.

Show the following properties:

[1] For details on LPs and the simplex algorithm(s) see any textbook on linear optimization, e. g. [HK00]

(i) At any time is $(S, V \setminus S)$ a (s,t)-cut.

(ii) The capacity of the cut $(S, V \setminus S)$ never increases during the run of the algorithm.

(iii) If f fulfills the flow conservation property, then the flow value $v(f)$ equals the capacity $c(S, V \setminus S)$ of the cut.

References

[HK00] Horst W. Hamacher and Kathrin Klamroth. *Lineare und Netzwerk-Optimierung / Linear and Network-Optimization.* Vieweg+Teubner Verlag, Wiesbaden, 2000. URL: `http://www.springerlink.com/index/10.1007/978-3-322-91579-5`.